RESEARCH AND APPLICATION OF NONLINEAR ANALYSIS METHOD IN STEEL STRUCTURES

Nghiên cứu và áp dụng phương pháp phân tích phi tuyến trong kết cấu thép

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Abstract: Modern computational technology is opening new avenues for optimization and material savings in structural design. Nevertheless, conventional design methods currently exhibit a distinct inconsistency. Specifically, while structural modeling and internal force analysis typically rely on linear elastic analysis (based on the assumption of small deformations), the load-bearing capacity check of structural members utilizes non-linear methods, as specified in the equations of current design standards. This discrepancy prevents mathematical models from accurately reflecting the true working state of structural systems.

To address this issue, this research focuses on applying non-linear analysis using the Finite Element Method (FEM). This approach yields two major benefits: accurately simulating the actual behavior of structures under complex loading conditions, and mitigating the limitations of conventional design methods. Notably, the reliability of the numerical simulations will be directly cross-referenced and validated with experimental results, establishing a solid foundation for optimization problems.

Tóm tắt: Công nghệ tính toán hiện đại đang mở ra hướng đi mới giúp tối ưu hóa và tiết kiệm vật liệu trong thiết kế công trình. Dù vậy, phương pháp thiết kế truyền thống hiện nay lại bộc lộ rõ sự thiếu đồng nhất. Cụ thể, quá trình mô hình hóa và phân tích nội lực kết cấu thường dùng phương pháp phân tích đàn hồi tuyến tính (phù hợp với giả định hệ kết cấu có biến dạng nhỏ), nhưng đến bước kiểm tra khả năng chịu lực của các cấu kiện lại áp dụng phương pháp phi tuyến được thể hiện trong các phương trình của tiêu chuẩn hiện hành. Sự thiếu đồng nhất này khiến mô hình toán học chưa phản ánh chính xác nhất trạng thái làm việc thực tế của hệ kết cấu.

Để làm rõ vấn đề này, nghiên cứu mới đã tập trung ứng dụng phân tích phi tuyến trong phương pháp phần tử hữu hạn (FEM). Giải pháp này mang lại hai lợi ích lớn: Mô phỏng chính xác ứng xử thực tế của kết cấu dưới các dạng tải trọng phức tạp; giảm thiểu sự hạn chế của phương pháp thiết kế truyền thống. Đặc biệt, độ tin cậy của mô phỏng số sẽ được đối chiếu và kiểm chứng trực tiếp bằng các kết quả thí nghiệm thực tế, tạo cơ sở vững chắc cho các bài toán tối ưu.

Keywords: Linear elastic analysis, non-linear analysis, structural behavior, optimization.

Từ khóa: Phân tích đàn hồi tuyến tính, phân tích phi tuyến tính, ứng xử của hệ kết cấu, tối ưu hóa.

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I. LITERATURE REVIEW

1.1. Linear Analysis

Linear analysis is based on three primary assumptions:

1.1.1. Small Deformation

- Displacements are small relative to the structural dimensions and do not affect the initial geometry.

- The axis and inclination angle of the elements remain virtually unchanged.

1.1.2. Linear Stress-Strain Relationship

Hooke's Law describes the linear relationship between the stress and strain of an elastic material when subjected to loads within its elastic limit.

$\sigma = E.\varepsilon$ (1)

Where:

σ: Stress, unit in Pa (N/m^2),

ε: Strain, dimensionless,

E: Modulus of elasticity (Young's modulus), characterizing material stiffness, unit in Pa (N/m^2).

1.1.3. Equilibrium established on the initial geometry

Equilibrium conditions are formulated based on the initial geometry of the structure. Loads and reactions are calculated on the un-deformed structure.

1.1.4. Fundamental equation

The matrix form of the equilibrium equation:

$$K.u=P \quad (2)$$

Where:

K: Stiffness matrix (constant during the analysis),

u: Nodal displacement vector,

P: External load vector.

1.2. Nonlinear Analysis.

Nonlinear analysis is a structural analysis method where the relationship between load and displacement is no longer linear, due to the effects of large deformations, nonlinear material behavior (plasticity, cracking, softening), or changing boundary conditions (contact, sliding, etc.).

There are three main types of nonlinearity:

1.2.1. Geometric Nonlinearity

Nature:

- Occurs when large displacements significantly alter the initial geometry.

- Equilibrium equations must be formulated based on the actual deformed configuration of the structural system (not the initial geometry).

Manifestations:

- P–Δ effect (global frame) and P–δ effect (local element).

- Leads to geometric instability (buckling).

1.2.2. Material Nonlinearity

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Nature:

- Occurs when the material exceeds its elastic limit (yield point).

- Stress is no longer linearly proportional to strain (nonlinear σ–ε relationship).

Manifestations:

- Includes: Elastic-plastic behavior; tensile cracking and compression crushing (softening); creep; shrinkage, etc.

1.2.3. Contact Nonlinearity

Nature:

- The connection between two surfaces can change during operation (contact opening and closing).

- Causes abrupt changes in boundary conditions.

1.2.4. General equation of nonlinear problems.

General equilibrium equation:

$$R(u) = P - F(u) = 0 \quad (3)$$

R(u): Residual vector (out-of-balance force vector).

P: External load vector.

F(u): Internal force vector, dependent on displacement.

Cannot be solved directly ⇨ requires iterative methods.

II. RESEARCH METHODOLOGY

2.1. Experimental Model

The experimental model is a welded built-up steel plate girder structure reinforced with transverse stiffeners. The cross-section and geometric dimensions of the steel girders are illustrated in Figures 1 and 2.

Figure 1. Actual photographic illustration of the experimental model.

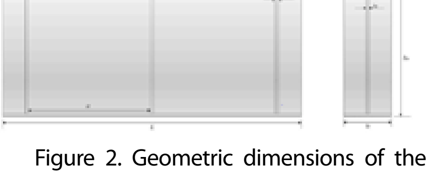


Figure 2. Geometric dimensions of the model.

The experimental models are constrained to fixed section heights of H500, H600, and H700, with detailed dimensional parameters listed in Table 1.

Table 1. Summary table for geometry of the experimental models

Experiment sample symbol	Steel grade	L (mm)	hw (mm)	a (mm)	tw (mm)	tf (mm)	ts (mm)	a/hw	wo (mm)
V304D-H500-ad1.0	QN1803	1200.7	499.3	498.5	3.67	11.76	12.07	1.0	3.1
V304D-H500-ad1.0-R	QN1803	1200.1	498.3	499.5	3.67	11.76	12.07	1.0	3.1
V304D-H600-ad1.0	QN1803	1400.3	598.6	599.5	3.67	11.78	12.1	1.0	5.4
V304D-H700-ad1.0	QN1803	1600.1	697.2	699.3	3.68	11.81	11.98	1.0	5.3

2.2. Testing and Analysis Methods

This paper focuses on experiments conducted on built-up plate girders to determine the shear buckling strength of plate girders fabricated from QN1803 steel.

Prior to conducting the shear tests, the initial geometric imperfections of the girders were measured using a 3D scanning machine. Additionally, a series of tensile coupon tests were performed to investigate the influence of the steel rolling direction on the mechanical properties of the material.

The tensile coupons were prepared and tested in three different directions: longitudinal, diagonal, and transverse, to accurately determine the strength of the QN1803 steel material.

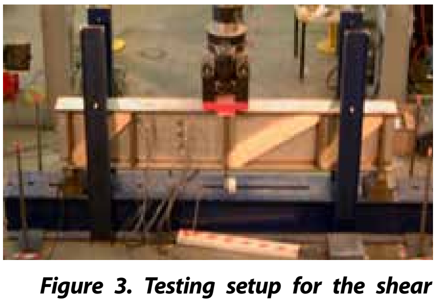


Figure 3. Testing setup for the shear buckling strength of plate girders.

Subsequently, a Finite Element Method (FEM) model, accounting for material nonlinearity and initial geometric imperfections, was developed and validated against the experimental results.

Finally, the current design formulas used to determine the local shear buckling strength of plate girders, as presented in the Eurocode (EN 1993-1-4) [1], are evaluated

2.3. Computational Model

The Abaqus software was utilized to construct the computational model based on the actual geometric parameters of the test specimens.

Theoretical boundary conditions were applied to the model, reflecting the actual working state of the girder components when installed in the test setup. These constraints restrict translations in specific degrees of freedom and define the applied displacement conditions.

A material nonlinear analysis method was applied, explicitly incorporating the manufacturing imperfections of the test specimens. This approach was used to analyze the instability and geometric imperfections of the elements, thereby determining the ultimate limit state of the girders under consideration.

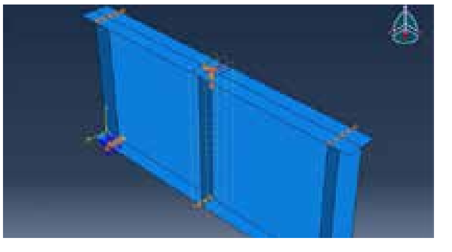


Figure 4. Simulation model of steel beam in Abaqus

III. RESULTS ANALYSIS

After analyzing the simulation models of the steel girder structure with stiffeners using Abaqus software, the following models were evaluated:

- MHTN: The actual experimental model.

- MHPT_VL: The simulated experimental model accounting for both geometric nonlinearity (due to the manufacturing imperfections of the girder) and material nonlinearity (based on material strength parameters recorded during tensile coupon tests).

- MHPT_EC: The simulated experimental model based on the Eurocode standard (EN 1993-1-4+A1), which accounts for manufacturing imperfections and material strength parameters calculated according to the code's formulas.

Based on the load-displacement (shear force-deflection) curve, the measuring equipment recorded the load (shear force) and displacement (deflection) of the experimental girder model via a reference point located at the middle of the top flange:

Considering the experimental model V304D-H500-ad1.0, which has a fixed height of approximately H500, there is a minor discrepancy in the ultimate shear force of 1.078% and 1.16% between the experimental model (MHTN, with a maximum shear force of 688.04 kN) and the two simulation models (MHPT_VL and MHPT_EC, with maximum shear forces of 695.461 kN and 696.019 kN, respectively). Therefore, it is evident that the ultimate shear force analysis results of these three models are nearly identical. Furthermore, their corresponding displacements are also in close agreement, with an expected deflection of approximately 10 mm.

Further examining the simulation results for the V304D-H500-ad1.0-R model, which incorporates additional stiffeners at both ends of the girder, the maximum shear force in the experimental model (MHTN) reached 789.57 kN. This represents an increase of approximately 14.75% compared to the V304D-H500-ad1.0 model (688.04 kN). Similarly, when compared with the analysis models (MHPT_VL and MHPT_EC), the deviation remains marginal. Notably, the three curves almost perfectly coincide from 0 mm to 30 mm of displacement. Subsequently, the two simulation models (MHPT-VL and MHPT-EC) exhibit a downward trend in shear force as displacement progresses from 30 mm to 50 mm, whereas the experimental model (MHTN) maintains its shear force stability.

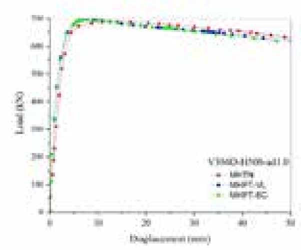
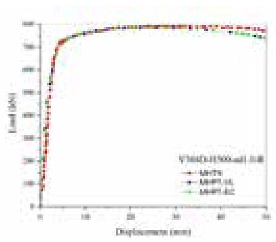


Figure 5. Load-displacement relationship graph for the V304-H500-ad1.0 model



Hình 6. Load-displacement relationship graph for the V304-H500-ad1.0-R model

Analyzing the graphs for the two experimental models V304D-H600-ad1.0 and V304-H700-ad1.0, the deviation between the maximum shear force curves for the three models (MHTN, MHPT-VL và MHPT-EC) is relatively distinct. Specifically, for the V304D-H600-ad1.0 model, the simulation models (MHPT-VL, MHPT-EC) are consistent during the linear elastic phase; however, a clear separation among the three models occurs upon transitioning into the yielding and strain-hardening phases. For the V304-H700-ad1.0 model, the two simulation models (MHPT-VL, MHPT-EC) align perfectly during the material's linear elastic phase. Yet, as the behavior shifts into the yielding, strain-hardening, and ultimate strength phases, the separation among the three analysis models becomes more pronounced, although the actual difference in shear force values remains insignificant.

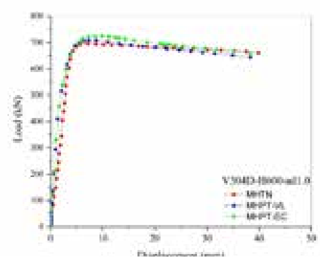


Figure 7. Load-displacement relationship graph for the V304-H600-ad1.0 model

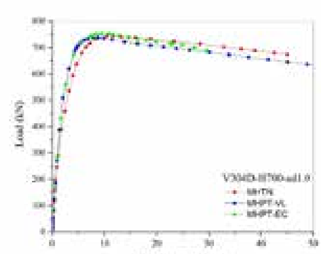


Figure 8. Load-displacement relationship graph for the V304-H700-ad1.0 model

IV. CONCLUSIONS AND RECOMMENDATIONS

Based on the simulation results of built-up steel plate girders using material nonlinear analysis, coupled with the effects of geometric nonlinearity (induced by fabrication imperfections), this study conducted a comprehensive comparison with experimental data.

For the girder specimens with a fixed height of H = 500 mm (specifically V304-H500-ad1.0 and V304-H500-ad1.0-R), the results from the simulation analysis and the code-based calculations exhibited a very high correlation with the experimental model. However, as the girder height increased in specimens V304-H600-ad1.0 and V304-H700-ad1.0, the discrepancies among the three analysis models became more pronounced. Nevertheless, this deviation remains within the permissible range and does not significantly affect the evaluation of the overall shear capacity of the structural members. The underlying causes of these variations warrant further investigation in subsequent in-depth studies.

From the above results, it can be affirmed that the application of nonlinear analysis to complex steel structural systems is a practical and urgent necessity. Rather than being limited to isolated components, leveraging the power of current algorithms and modern computational technology facilitates an accurate representation of the actual working state of the structure. This not only optimizes design and ensures material savings but also provides a highly reliable foundation for engineers to apply in the calculation and design of real-world construction projects.

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